2. V. S. Komel'kov, A. P. Kuznetsov, et al., "Dynamics of a plasma shell with an external current," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1982).
3. V. S. Komel'kov and V. I. Modzolevskii, "A coaxial accelerator for dense plasma," Fiz. Plazmy, 3, No. 5 (1977).
4. V. I. Vasil'ev, V. S. Komel'kov, et al., "A stable dynamic current filament," Zh. Tekh. Fiz., 30, 756 (1960).
5. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Academic Press, New York (1967).
6. S. A. Colgate and M. H. Jonson, "Hydrodynamic origin of cosmic rays," Phys. Rev. Lett., 5, No. 6 (1960).
7. N. G. Basov, S. V. Bobashev, et al., "Spectrometric observations of multiply charged ions of megavolt energies in a laser plasma," Pis'ma Zh. Eksp. Teor. Fiz., 36, No. 7 (1982).

MAGNETOPLASTICITY OF MAGNETIZABLE MEDIA

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An important problem in modern mechanics is the study of the behavior of continuous media in strong electromagnetic fields [1]. Here media with strongly manifested magnetic properties are of great interest, because in such media the interaction of the medium with the electromagnetic field through the ponderomotive forces and energy flow from the field into the material, owing to the magnetization, plays an important role.

Magnetoplastic flows of magnetizable media are realized in elements of engineering structures, operating in magnetic fields, which create pressures close to the yield point of the magnetic material [2]. The study of such flows is of interest for powder metallurgy and high-pressure metal working [3]. It is expected that the magnetization effect is important in materials formed by sintering of ferromagnets with other metals.

The basic system of equations describing magnetoplasticity, ignoring the effects of magnetization, is derived in [4]. The basic equations of motion of magnetizable media are derived in [5].

We assume that the processes studied in the incompressible and perfectly conducting medium are quasistationary and that the magnetization is reversible. To describe plastic flows of magnetizable media in an electromagnetic field, starting from [4, 5] we obtain a closed system of equations in the following form:

$$\nabla_k v^k = 0; \tag{1}$$

$$\rho\, dv^i/dt = \nabla_k p^{ik}; \tag{2}$$

$$\rho T \frac{d}{dt}\left(s + \frac{\mu_0}{\rho}\int_0^H \left(\frac{\partial M}{\partial T}\right)_{\rho,H} dH\right) = P^{0*}_{ik} v^{ik} + \operatorname{div}(\lambda \nabla T); \tag{3}$$

$$\nabla_k B^k = 0; \tag{4}$$

$$\partial \mathbf{B}/\partial t = \operatorname{curl}[\mathbf{v},\ \mathbf{B}]; \tag{5}$$

$$P^{0*}_{ik} = \eta v_{ik}; \tag{6}$$

$$\frac{1}{2} P^{0*}_{ik} P^{0*ik} = k^2. \tag{7}$$

Here $P_{ik} = \overset{0}{P}_{ik} + \overset{H}{P}_{ik}$ is the total stress tensor, and $P^H_{ik} = -g_{ik}\left(\mu_0 \frac{H^2}{2} + \mu_0 \int_0^H \left(M - \rho\left(\frac{\partial M}{\partial \rho}\right)_{T,H}\right) dH\right) + H_i B_k$ is its magnetic part; ρ, v, T, and s are the density, velocity, temperature, and entropy of the medium, respectively; **H** and **B** are the magnetic field intensity and induction; λ is the coefficient of the thermal conductivity; the asterisk indicates the divisor of the stress

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tensor of the medium $P^{0*}_{ik} = P^0_{ik} - \frac{1}{3} g_{ik} P^{0l}_l$; $\eta = \frac{k\sqrt{2}}{\sqrt{v_{ik}v^{ik}}}$; and $v_{ik} = (1/2)(\nabla_i v_k + \nabla_k v_i)$ is the tensor of the deformation velocities. The magnetization law is given in the general form $\mathbf{M} = M(\rho, T, H)\mathbf{H}/H$. In this case, when the magnetization does not depend on the temperature $M = M(\rho, H)$, the equation describing the change in the entropy decouples from the basic system (1)-(7) and serves as a means for determining the temperature field after the velocity and magnetic field are determined.

As an example, we shall study the problem of the magnetoplastic flow of a tube (the inner and outer radii of the tube are a and b) in the presence of a frozen-in field under the action of the inner and outer pressure. As usual in the theory of plastic flows [6], we shall neglect the time derivatives and the convective terms in the equation of motion (2). We assume that the material is magnetized to saturation ($M = m_0 = \text{const}$). From symmetry considerations, in a cylindrical system of coordinates (r, φ, z) all quantities depend only on r and, in addition, $P^0_{r\varphi} = P^0_{rz} = P^0_{\varphi z}$ and $v_\varphi = v_z = 0$. To determine the components of the stress tensor, we have

$$\frac{d}{dr} P^0_{rr} \pm \frac{P^0_{rr} - P^0_{\varphi\varphi}}{r} - B_r \frac{dH_r}{dr} - B_\varphi \frac{dH_\varphi}{dr} - B_z \frac{dH_z}{dr} + \frac{d}{dr} B_r H_r + \frac{B_r H_r - B_\varphi H_\varphi}{r} = 0,$$

$$P^0_{\varphi\varphi} = P^0_{rr} \mp 2k, \quad P^0_{zz} = P^0_{rr} \mp k.$$

The sign in these equations is chosen in accordance with the geometry of the problem.

Let the frozen-in magnetic field $\mathbf{H}_0$ and the magnetic field $\mathbf{H}$ in the cavity of the tube, under whose action the plastic flow occurs, have the form $\mathbf{H}_0(0, H_{\varphi 0}, H_{z0})$ and $\mathbf{H}(0, H_\varphi, H_{z0}(a))$. Assuming that the pressure on the outer surface of the tube is equal to zero, we have the boundary condition $P^0_{rr}(b) = 0$. We set $t = a/b$ and $q = H_{z0}(a)/H_{\varphi 0}(a)$. Then the magnitude of the magnetic field $H_\varphi(a)$ at which the plastic flow begins satisfies the relation:

$$\mu_0 \frac{H^2_\varphi(a)}{2} = 2k \ln \frac{b}{a} + \mu_0 \frac{H^2_{\varphi 0}(a)}{a^2} \left(b^2 - \frac{a^2}{2}\right) + 2\mu_0 m_0 \frac{b^2}{a^2} H_{\varphi 0}(a)$$
$$\times \left(\frac{1+q^2t^2}{\sqrt{1+q^2}} - \frac{t}{2}\sqrt{1+q^2t^2}\right) + \mu_0 m_0^2 \frac{b^2}{a^2}\left(\frac{1+q^2t^2}{1+q^2} - t\sqrt{\frac{1+q^2t^2}{1+q^2}}\right). \quad (8)$$

When there is no frozen-in field, the flow of the tube begins when $\mu_0 H^2_\varphi(a)/2 = 2k\ln(b/a)$. Since in (8) the expressions in parentheses are always positive, the frozen-in field always prevents the flow. A magnetization $m_0 > 0$ amplifies this effect of the field.

We shall study the flow of a tube under the action of an external magnetic pressure. The boundary condition is $P^0_{rr}(a) = 0$. Let the frozen-in field $\mathbf{H}_0$ and the external field $\mathbf{H}$ have only transverse components, i.e., $\mathbf{H}_0(0, H_{\varphi 0}, 0)$ and $\mathbf{H}(0, H_\varphi, 0)$. Setting $H_{\varphi 0}(b) = n$, we obtain for the magnitude of the external field at which the plastic flow begins the equation

$$\mu_0 H^2_\varphi(b)/2 = 2k \ln(b/a) - f_{m0}(t, n),$$

where

$$f_{m0}(t, n) = \frac{\mu_0}{2}\left((1 - 2t^2)n^2 + 2m_0 t(1 - 2t)n + 2m_0^2 t(1 - t)\right).$$

To determine the frozen-in fields that induce or prevent flow, it is necessary to study the sign of $f_{m0}(t, n)$. Graphs of the function $f_{m0}(t, n)$ for different values of the thickness parameter t are shown by the solid lines in Figs. 1 and 2, where the following cases are presented: $0 < t < 1/2$, $t = 1/2$, $1/2 < t < 2/3$ (Fig. 1a-c, respectively); $t = 2/3$, $2/3 \leq t < 1/\sqrt{2}$ (Figs. 2a and b, respectively); the straight line in Fig. 2c corresponds to $t = 1/\sqrt{2}$, when $f_0(1/\sqrt{2}, n) = 0$, while the parabolic line corresponds to $-1/\sqrt{2} < t < 1$. Figures 1 and 2 show for comparison graphs of the function $f_0(t, n)$ (broken lines) for a nonmagnetic medium with different t. We note the existence of the singular points $n_0 = m_0(1 - t)/(2t - 1)$, $n_1 = m_0 t(2t - 1)/(1 - 2t^2)$, $n_2 = 2m_0$, $n_{3,4} = m_0((2t - 1)t \mp \sqrt{t(3t - 2)})/(1 - 2t^2)$, $n_5 = m_0/\sqrt{2}$ and values of the function

$$f_{m0}(1/2, n) = f_0(1/2, n) + \mu_0 m_0^2/4 = \frac{\mu_0}{4}(n^2 + m_0^2),$$
$$f_{m0}(t, n_1) = \mu_0 m_0^2 t(2 - 3t)/2(1 - 2t^2),$$

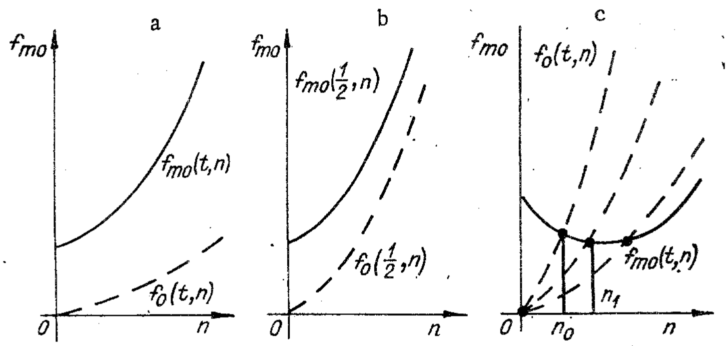

Fig. 1

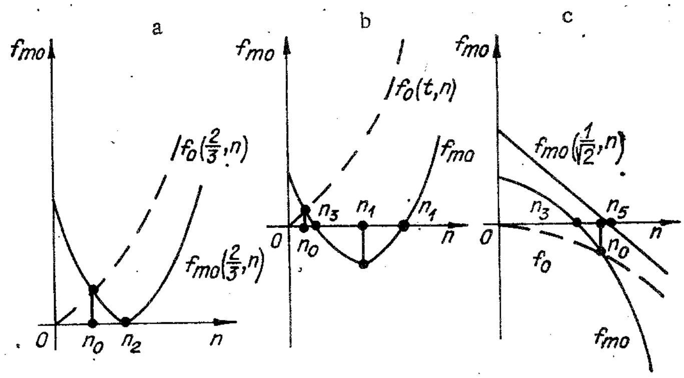

Fig. 2

which play an important role in the analysis of the results obtained. We have the following cases: for ($2/3 < t < 1/\sqrt{2}$, $n_3 < n < n_4$) as well as for ($t = 1/\sqrt{2}$, $n > n_5$) and ($1/\sqrt{2} < t < 1$, $n > n_3$) the frozen-in magnetic field prevents plastic flow.

For ($t = 2/3$, $n = n_2$), ($2/3 < t < 1/\sqrt{2}$, $n = n_3$ or $n = n_4$), ($t = 1/\sqrt{2}$, $n = n_5$), and ($1/\sqrt{2} < t < 1$, $n = n_3$) the frozen-in field does not affect the magnitude of $\mu_0 H_\varphi^2(b)/2$.

For all the other changes in t and n, the frozen-in field induces plastic flow.

To complete the analysis, we must add the fact that for ($1/2 < t < 1$, $n = n_0$) magnetization does not affect the process, while for ($1/2 < t < 2/3$, $n = n_1$) the flow-inducing effect of the frozen-in field has a minimum, just as in the case when for ($2/3 < t < 1/\sqrt{2}$, $n = n_1$) the flow-preventing effect has a maximum.

We note that the existence of values of n_0–n_5 was discovered only because of the fact that the magnetizability of the material was taken into account.

Let the frozen-in field $\mathbf{H}_0$ and the external field $\mathbf{H}$ have the form $\mathbf{H}_0(0, H_{\varphi 0}, 0)$ and $\mathbf{H}(0, H_{\varphi 0}(b)r/b, H_z)$. Then for the value of the external longitudinal field at which flow begins we have

$$\mu_0 \frac{H_z^2(b)}{2} = 2k \ln \frac{b}{a} - g_{m0}(t, n),$$

where

$$g_{m0}(t, n) = \mu_0 \left((1 - t^2) n^2 + m_0 t (1 - 2t) n + m_0^2 t (1 - t)\right).$$

The critical values of the field in this case are equal to

$$n_0 = \frac{m_0 (1 - t)}{2t - 1}, \quad n_1 = \frac{m_0 t (2t - 1)}{2(1 - t^2)},$$

$$n_2 = \frac{2}{3} m_0, \quad n_{3,4} = \frac{m_0 \left((2t - 1) t \mp \sqrt{t(5t - 4)}\right)}{2(1 - t^2)},$$

$$g_{m0}\left(\frac{1}{2}, n\right) = \frac{\mu_0}{4}\left(3n^2 + m_0^2\right), \quad g_{m0}(t, n_1) = \frac{\mu_0 m_0^2 t (4 - 5t)}{4(1 - t^2)}.$$

We have the following possibilities:

for $(4/5 < t < 1, n_3 < n < n_4)$ the frozen-in field prevents flow;

for $(t = 4/5, n = n_2)$ and $(4/5 < t < 1, n = n_3$ or $n_4)$ the frozen-in field does not affect the flow; and

for other values of t and n the frozen-in field gives rise to plastic flow.

Replacing in Figs. 1 and 2a and b $f_{m0}(t, n)$ by $g_{m0}(t, n)$ and 2/3 by 4/5, we obtain the qualitative picture of the change in the function $g_{m0}(t, n)$ for all $t(0 < t < 1)$ and $n(n > 0)$ and we can follow the effect of the frozen-in field on the flow just as was done in the preceding example.

The critical values n_0–n_4 were discovered only because of the fact that the magnetizability of the material was taken into account.

The studies performed show that the magnetic properties of the medium substantially affect the nature of the plastic flow. Here, depending on the intensity of the applied magnetic field, the magnetization can both induce and prevent plastic flow. For typical materials (commercial iron, soft steel, and nickel) the state of saturation, where the law $M = m_0$ can be used, appears in quite weak fields $n_k \sim 400$ Oe. The corresponding regions where the frozen-in field induces flow become insignificantly narrower than the regions shown in the graphs, and an additional restriction $t \leqslant t_k$ is imposed on the parameter t, where for the materials under study $t_k = 0.94$–0.98. The analysis of the flow for $n < n_k$, when saturation of the magnetization is not achieved and magnetocaloric effect play an important role, requires additional studies.

In conclusion, we note that in the limit of nonmagnetic medium, our results coincide with the results in [4].

LITERATURE CITED

1. L. I. Sedov, "Promising directions and problems in the mechanics of continuous media," Prikl. Mat. Mekh., 40, No. 6 (1976).
2. G. Kol'm and A. Frimen, "Strong magnetic fields," Usp. Fiz. Nauk, 88, No. 4 (1966).
3. I. V. Belyi, S. M. Fertik, and A. T. Khimenko, Handbook on Magnetic Pulse Working of Metals [in Russian], Vishcha Shkola, Kharkov (1977).
4. V. P. Demutskii and R. V. Polovin, "Magnetoplastic flow," Prikl. Mat. Mekh., 33, No. 5 (1969).
5. I. E. Tarapov, "Hydrodynamics of polarizable and magnetizable media," Magn. Gidrodin., No. 1 (1972).
6. V. V. Sokolovskii, Theory of Plasticity [in Russian], Vysshaya Shkola, Moscow (1969).